Hypersphere space-time model

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Abstract

The origin of the three spatial dimensions as well as that of time is deduced from fundamental principles (symmetry). The structure resulting from this construction looks like an hypersphere of which each energy particle constitutes a dimension, forming a loop or a string covering the whole universe. This model shall be linked to the existing theories that are in adequation with the experience.

Article

Space

Nothing (symmetry) generating something (energy) can be expressed by the addition and the multiplication of an energy quantum (a) and its opposite ($\bar{a}$) :

$a + \bar{a} = 0$ (symmetry), $a\bar{a} = 1$ (energy) $\rightarrow$ $a = i$ and $\bar{a} = -i$ where $i^2 = -1$.

The quantum (a) is a complex number ($a = a_1+a_2 i \in \mathbb{C}$, $a_1, a_2 \in \mathbb{R}$, $i^2 = -1$) so it behaves like a wave, more precisely like the π/2 phase of a virtual (potential) standing wave covering the whole universe. It's the same for the opposite ($\bar{a}$).

The quantum (a) and its opposite ($\bar{a}$) form a pair of complex numbers ($a,\bar{a}$). These two elements on their own dimension are linked together thanks to a new dimension thanks to the external or vectorial product $\wedge$: $a\wedge\bar{a} = v$. The vector (v) exists in a three dimensional space ($a,\bar{a},a\wedge\bar{a}$) that can be represented by a quaternion $q \in \mathbb{H}$

$$q = s + (v) = s + x_1 i_1 + x_2 i_2 + x_3 i_3$$

where $s, x_1, x_2, x_3 \in \mathbb{R}$, $i_1^2 = i_2^2 = i_3^2 = i_1 i_2 i_3 = -1$, $(v) = (x_1,x_2,x_3)$

more precisely by a vectorial quaternion where s = 0. The quaternion is a piece of momentum, a piece of energy. Energy conservation implies that the result of the operation on two pairs of quanta shall not be null, which is in adequation with the quaternionic multiplication. So a fundamental element can be represented by a quaternion. It's the Hamilton's dream.

Considering that the universe is made of such fundamental energy elements, energy conservation implies a constant (finite) number of such elements or else an homogeneity that means that all elements are identical or even both, constant number and homogeneity. Homogeneity seems to be a principle and it will be supposed. The easiest way to explain homogeneity is to have only one element that interacts with itself in several ways but this extreme hypothesis will require further study.

According to the homogeneity principle, all quaternions can be considered as unitary quaternion (u) of norm 1 ($||u|| = 1$), following the definition

$$||q||^2 = \langle q|q\rangle = q\bar{q} = s^2+x_1^2+x_2^2+x_3^2$$

where $||q||$ is the norm, $\langle q|q'\rangle = q\bar{q}'$ is the bra-ket product and $\bar{q} = s - x_1 i_1 - x_2 i_2 - x_3 i_3$ is the conjugate of (q). Note that $\bar{q}/||q||^2$ is the inverse of any $q \neq 0$ and that the bra-ket product acts like a (right) division, not commutative and even not associative in general.

Each fundamental energy element has - or "is" because there is no other characteristic - its own three dimensional (3D) space, perfectly in accordance with the special relativity. Independancy of elements means that there is an orthogonal representation of them. Each element is on its own dimension, forming an hypersphere of 3D spaces, an hypersphere of quaternions.

Mass

In a first approach, mass existence could be linked to a non negative value of (s) in the quaternion. But this approach doesn't answer to a lot of questions.

Mass is specific to some particles, to some set of energy, not to all kind of energy. Mass is expressed inside the Dirac equation. Mass comes from external interaction with the BEH (Higgs) boson, this interaction changes the chirality of the particle. Mass is constant at rest and is spread in 3 generations, according to some relations (CKM and PMNS matrices). Mass is annihilated by the antiparticle. All these different things shall match. The following explores some research areas but it needs to be developed deeper and more rigorously.

Biquaternion

The square of a vectorial quaternion (v) is a negative number

$$v^2 = (x_1 i_1 + x_2 i_2 + x_3 i_3)(x_1 i_1 + x_2 i_2 + x_3 i_3) = -(x_1^2 + x_2^2 + x_3^2)$$

because $i_j i_j = -1$, $i_j i_k + i_k i_j = 0$, $j \neq k$

Using an imaginary vector ($v^* = vi$, $i^2 = -1$) instead of (v) implies a positive square $v^{*2} \geq 0$ which is more in adequation with the real world. Such structure is called a bivector and it seems to be the fundamental structure as seen below.

Unitary quaternion can be expressed thanks to the SU(2) Lie group generators $i\sigma_j$ j = 1,2,3 where σ_j are the Pauli matrices which are completed by the unit matrix σ_0

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbb{1} \qquad \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

These matrices can generate by multiplication

- a scalar (s), similar to a real number : $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \mathbb{1}$
- a vectorial quaternion (v), similiar to i_j : $\sigma_1\sigma_2 = i\sigma_3 \equiv i_3$, $\sigma_2\sigma_3 = i\sigma_1 \equiv i_1$, $\sigma_3\sigma_1 = i\sigma_2 \equiv i_2$
- a bivector (v^*), similar to $i_j i$: $i_1 i \equiv -\sigma_1$, $i_2 i \equiv -\sigma_2$, $i_3 i \equiv -\sigma_3$
- a pseudo-scalar (s^*), similar to the imaginary number i : $\sigma_1\sigma_2\sigma_3 = \mathbb{1}i$

By addition and multiplication, these matrices can generate a biquaternion (b) which is the sum of a real quaternion (q) and an imaginary quaternion (q')

$$b = (s + v) + (s^* + v^*) = (s + v) + (s' + v')i = q + q'i$$

where $b \in \mathbb{B}$, $q, q' \in \mathbb{H}$, $s^* = s'i$, $v^* = v'i$

which is also a quaternion with complex numbers instead of real numbers

$$b = z_0 + z_1 i_1 + z_2 i_2 + z_3 i_3 \qquad \text{where } z_j \in \mathbb{C},\ i i_j = i_j i$$

The definition of the conjugate of (b) is similar to the one of the quarternion

$$\bar{b} = z_0 - z_1 i_1 - z_2 i_2 - z_3 i_3$$

As the quaternion, the multiplication of two biquaternions is a biquaternion. The biquaternion has a norm $||b||^2 = ||q||^2 + ||q'||^2$ which is euclidian. Unlike the quaternion, a biquaternion (b) has no inverse when [1]

$$b = q(1+\sigma) \qquad \text{where } q \in \mathbb{H} \text{ is a quaternion and } \sigma \text{ is a unitary bivector } (\sigma^2 = 1)$$

$$b = (s'+\sigma'i)(1+\sigma) = s'+\sigma'i-\sigma'\sigma + \sigma's$$

thus because $\overline{\sigma} = -\sigma$ and if $b=b_1b_2$ then $\overline{b} = \overline{b}_2\overline{b}_1$

$$b\overline{b} = q\,(1+\sigma)\,\overline{(1+\sigma)}\,\overline{q} = q\,(1+\sigma)\,(1-\sigma)\,\overline{q} = q\,(1-1)\,\overline{q} = 0.$$

Dirac equation

The Dirac equation is

$$\left[i\hbar \frac{\partial}{\partial t} \right] \psi = \left[mc^2 \alpha_0 - i\hbar c\, \alpha_j \frac{\partial}{\partial x_j} \right] \psi$$

$$\alpha_0 = \begin{bmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{bmatrix} \qquad \alpha_j = \begin{bmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{bmatrix}$$

By setting $p = mc$ and $\varphi = \psi\, e^{ip^2/(2\hbar)}$, the equation becomes

$$\left[i\hbar \frac{\partial}{\partial t} + i\hbar c\, \alpha_0 \frac{\partial}{\partial p} + i\hbar c\, \alpha_j \frac{\partial}{\partial x_j} \right] \varphi = 0$$

The mass is processed in a similar way as the moment.

Any biquaternion (φ) can be split in two spinors φ_L and φ_R that are also biquaternions

$$\varphi = \varphi_L + \varphi_R = \left[\frac{1}{2} \varphi(1+\sigma_j) \right] + \left[\frac{1}{2} \varphi(1-\sigma_j) \right]$$

where σ_j is a bivector and $\sigma_j^2 = 1$. These spinors have the following characteristics

$$\varphi_L\sigma_j = \varphi_L \text{ and } \varphi_R\sigma_j = -\varphi_R$$

so they are a solution of the Dirac equation [2] as particle and antiparticle. As seen above, φ_L and φ_R have no inverse.

In an highly speculative manner, some conclusions could be done. A particle is a spinor which is a biquaternion without inverse, that can explain why a particle exists. The mass of the particle is the same as the momentum, by nature and by quantity. The mass is the mirror of the momentum, they are undistinguishable and they can therefore be interchanged. The momentum has three dimensions, which is then the same for the mass, which explains the three generations of mass. The mass is constant as well as the momentum, rest is an illusion due to the alternation of mass and momentum, that changes the chirality and the direction.

SU(3) Lie group

The SU(2) Lie group is homomophic to the SO(3) Lie group. SO(3) generators are

$$J_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad J_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad J_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

and *i*SO(3) generators can be defined by

$$I_1 = \begin{bmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad I_2 = \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \quad I_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

because the determinant is 1 but $I_j^2 = -1$.

The multiplication of the matrices as well as the unit matrix by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

gives the Gell-Mann matrices and the corresponding SU(3) Lie group generators. So there is a link between a biquaternion and the SU(3) Lie group. **what about product = 0 ?**

Interaction

Each element has its own 3D space but there is apparently a common 3D space. This common space is a space of interactions. An interaction is a 3D space-time relation between particles, they share the same 3D space-time, they project themselves on the space. We can imagine different common spaces, a bit like multiple universes, but there are restrictions.

The standard model of interactions U1×SU2×SU3 can be generated by quaternions.

An interaction is a kind of division in the set of biquaternions.

A particle can be seen as a punctual projection of space-mass on the common 3D space.

The common 3D space is euclidian, so flat, because the norm of biquaternions is euclidian.

Entanglement has an obvious solution here. The entangled particles share the same 3D space but are outside the "common" (our) 3D space. They propagate in their own 3D space but during interaction, their space combines with the common one, which can randomly be done in several different ways.

Time

Time is just the result of space and mass propagation, expressed by the norm as the Minkowski's formula $s^2+x^2+y^2+z^2 = c^2t^2$. Time is relative to interaction. Time increases entropy because number of changes increases by time.

Conclusion

Based on the hope that Nature is simple, this article introduces a new representation of space-time structure of the universe : an hypersphere structure on a multi-dimensional space, each dimension is an energy quantum with its opposite forming a biquaternion covering the whole universe. The implications of this hypothesis are vast and go far beyond this short article.

There is still a long way to envolve the whole physic in one theory but this bottom-up approach, from simple principles to more complex structures, in adequation with the observed reality, is probably a good way to elaborate a simple and comprehensive theory. This intuitive approach tries to answer to a fundamental question : why has the universe an apparent three dimensional structure in addition of time, which is far from an evidence ?

Whether the theory is correct or not, it seems increasingly clear that the visible common space-time is not a fundamental structure, it's the consequence of the interaction between particles. That's why calculations based only on our visible space-time can become unstable.To explain the universe, the ether is not necessary and perhaps not space-time either.

References

[1] CASANOVA Gaston, "L'algèbre vectorielle", *Que sais-je?* n°1657 p49 (1976)

[2] ibidem p69

Puzzle

- énergie
  - moment = addition ?, énergie = multiplication
  - conservation du moment (d/dx=0) -> phase varie mais pas amplitude (norme)
- quaternion
  - déterminant de matrice = volume
  - quotient de vecteurs, bivecteur = matrice de Pauli,
  - matrice orthogonale → conjugué complexe (aussi matrice)
  - bra-ket = division, non associatif, indiscernable
  - moment (vecteur)
  - propagation (Minkowski, unitarité A -> $U^{-1}AU$)
  - SU2 -> SU3 via SU2xSU2xSU2=U3 ou SO3
- masse
  - antimatière pour masse négative (équation Dirac γ_0), symétrie masse/espace car pas d'inverse biquaternion (norme = 0), chiralité compense masse
  - propagation = 0 si masse = vitesse ?
  - temps/masse crée alternance chiralité
  - double couverture SU(2) de SO(3) crée masse ?
  - familles : rotation mais énergies différentes aussi
  - biquaternion (s, iv), sans inverse, i4 -> 1 (chapeau mexicain)
  - variable en théorie mais constante en pratique(aléatoire ? seule réaction possible ?)
  - brisure de symétrie -> probabilités asymétriques (pas 1/2)
- interaction
  - constante de structure (./. vitesses)
  - interaction ponctuelle → projection
  - matérialisation d'un choix par rapport à une référence
- temps
  - propagation par auto interaction (norme)
  - source de choix possibles
  - NB :
    - se baser sur c et h constants
    - imprécision/indétermination augmente avec le temps
    - exclusion de Pauli agrandit univers
    - projecteur $E^2=E$ -> E=0 ou 1
    - double converture SO3 par SU2 -> spin
    - boson virtuel si distance inférieure à fréquence
    - Lagrangien = onde car échange de valeurs cos/sin
    - où est la propriété de quantité de masse (interaction Higgs) ?
    - gravitation = rotation cône de lumière
    - interaction = brisure symétrie, de phase, d'onde stationnaire
    - intrication = pas orthogonal, dépendance linéaire
    - énergie empruntée de x dt à cause onde et déphasage
    - const cosmologique : force croit avec distance
    - temps et probabilité liés (improbable si temps court), temps flou, augmente entropie
    - action = mL^2/t
    - $d(\exp(m^2)f(x,t))/dm = 2m(\exp(m^2)f(x,t))$
    - $E^2 = m^2(c^2)^2 + p^2c^2 = (mc)^2c^2 + (mv)^2c^2$